TEST 2 **PURE MATHEMATICS 2320** FALL 2001

Name	**MUN Number**

Marks

[9] 1. Let $A = \{1, 2, 3, 4, 5, 10, 11, 12, 13, 20, 30, 40, 50\}$. Define the relation $\preceq$ on A by $a \preceq b$ if and only if $a \mid b$.

(a) Show that $(A, \preceq)$ is a poset.

(this question continues...)

(b) Draw the Hasse diagram for the poset.

(c) What are the

i. maximum element(s)?

ii. minimal element(s)?

iii. minimum element(s)?

iv. minimal element(s)?

[4] 2. Define $f : \mathbb{Z} \to \mathbb{Z}$ by $f : a \mapsto a^2 - 6a + 8$.

(a) Prove or disprove: f is injective.

(b) Prove or disprove: f is surjective.

[4] 3. Define $f : \mathbb{Z} \to \mathbb{Z}$, $g : \mathbb{Z} \to \mathbb{Z}$, and $h : \mathbb{Z} \to \mathbb{Z}$ by $f(x) = 2x^2 - x - 1$, $g(x) = x + 3$, and $h(x) = -2$. What are:

(a) $(f \circ g)(x)$?

(b) $(g \circ f)(x)$?

(c) $(f \circ g \circ h)(x)$?

(d) $(h \circ g \circ f)(x)$?

[6] 4. Prove that $A = (0, 1)$ and $B = (10, 15)$ have the same cardinality.

[7] 5. State whether each of the following sets is finite, countably infinite, or uncountable:

(a) $\mathbb{N} \cup \{-3, -2, -1, 0\}$

(b) $\emptyset$

(c) $3\mathbb{Z} + 2$

(d) $\{x \in \mathbb{Q} \mid x \in [1, 2]\}$

(e) $\{x \in \mathbb{Z} \mid x^2 + 2x - 8 \leq 0\}$

(f) $\{x \in \mathbb{R}, \mid x^2 + 2x - 8 \leq 0\}$

(g) $\{\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}\}$

[5] 6. Let $n \in \mathbb{N}$. Prove that $\sum_{i=1}^{n} i^3 = \left(\frac{(n)(n+1)}{2}\right)^2$.