MATH 4341 – Combinatorial Design Theory Assignment #4
Winter 2013

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 15:00 on Friday February 8th in Assignment Box #48.

1. Prove that there cannot exist a PBD$(v, \{k_1, k_2\}, \lambda)$ such that $k_1 < k_2$ and there is exactly one block of size k_1.

2. Let $t \geqslant 3$. Prove that if $\mathcal{D}$ is a t-design in which each block has size k and each set of t treatments occurs in λ_t blocks, then $\mathcal{D}$ is also a $(t-1)$-design. Determine the value of λ_{t-1}.

3. As part of the proof of Fisher's Inequality we relied on the fact that $\mathrm{rank}(AA^T) \leqslant \mathrm{rank}(A)$. Prove that this fact is true.

4. Suppose $\mathcal{D}$ is a BIBD(v, b, r, k, λ) with no repeated blocks. Let S denote the support of $\mathcal{D}$ and let $\mathcal{B}$ be the block set of $\mathcal{D}$. Let $\mathcal{T}$ denote the set of all k-subsets of S other than those k-subsets which are in $\mathcal{B}$. Prove that if $b < \binom{v}{k}$ then the sets S and $\mathcal{T}$ form a BIBD, and determine its parameters.

5. Suppose that B is a block of a BIBD(v, b, r, k, λ). Let x_i denote the number of blocks other than B that intersect B in precisely i elements. Prove that $\sum_{i=0}^{k} i x_i = k(r-1)$ and also that $\sum_{i=0}^{k} i(i-1)x_i = k(k-1)(\lambda - 1)$.