MATH 2320 – Discrete Mathematics   Assignment #3
Winter 2016

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Wednesday February 10th in Assignment Box #44.

1. Determine whether the relation $\mathcal{R}$ is reflexive:

    (a) $\mathcal{R} = \{(x, y) \in \mathbb{Z}^2 \mid x^2 + y^2 \text{ is odd}\}$

    (b) $\mathcal{R} = \{(x, y) \in \mathbb{Q}^2 \mid xy \geqslant 0\}$

2. Determine whether the relation $\mathcal{R}$ is symmetric:

    (a) $\mathcal{R} = \{(x, y) \in \mathbb{N}^2 \mid x + y = 10\}$

    (b) $\mathcal{R} = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^4 \geqslant 10\}$

3. Determine whether the relation $\mathcal{R}$ is antisymmetric:

    (a) $\mathcal{R} = \{(x, y) \in \mathbb{R}^2 \mid x \geqslant y\}$

    (b) $\mathcal{R} = \{(x, y) \in \mathbb{R}^2 \mid x^2 \geqslant y^2\}$

4. Determine whether the relation $\mathcal{R}$ is transitive:

    (a) $\mathcal{R} = \{(x, y) \in \mathbb{N}^2 \mid x + y = 10\}$

    (b) $\mathcal{R} = \{(x, y) \in \mathbb{Q}^2 \mid x + y \in \mathbb{Z}\}$

5. Define the relation $\sim$ on $\mathbb{R}^2$ by $(a, b) \sim (c, d)$ if $a + b = c + d$.

    (a) Prove that $\sim$ is an equivalence relation.

    (b) Provide a geometric description of $\overline{(2, 4)}$.

6. Let $A = \{2, 3, 4, \ldots, 17\}$ and define $\preceq$ on A by $a \preceq b$ if a divides b.

    (a) Draw the Hasse diagram for the poset $(A, \preceq)$.

    (b) Is $\preceq$ a total order?

    (c) Does this poset have a maximum? If yes, what is it?

    (d) Does this poset have a minimum? If yes, what is it?

    (e) Does this poset have any minimal elements? If yes, what are they?

    (f) Does this poset have any maximal elements? If yes, what are they?

    (g) What is the least upper bound of elements 3 and 4?

    (h) What is the greatest lower bound of elements 15 and 16?

7. Let $A = \{1, 2, 3, \ldots, 7\}$ and define the function $g : \mathcal{P}(A) \to \mathbb{Z}$ so that $g(x) = |x|$.

    (a) What is the domain of g?

    (b) How many elements are in the domain of g?

    (c) What is the range of g?

    (d) Is g surjective?

    (e) Is g injective?

    (f) Is g bijective?

8. Define $h : \mathbb{N}^2 \to \mathbb{N}$ by $h : (x, y) \mapsto (x + y)$.

    (a) State the range of h.

    (b) Is h surjective?

    (c) Is h injective?

    (d) Is h bijective?