

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Wednesday January 27th in Assignment Box #44.

1. Let a, b, u and v be integers such that $u \neq 0$ and $v \neq 0$.
Consider the statement P : If $au + bv = 0$ then $a = b = 0$.

- (a) Is P true? If yes, then prove P ; otherwise show that P is false.
- (b) State the contrapositive of P .
- (c) State the converse of P .
- (d) State the negation of P .

2. Let a_1, a_2, a_3 be positive integers and let $m = \prod_{i=1}^3 a_i$.

Prove that at least one of a_1, a_2, a_3 is at least $\sqrt[3]{m}$.

3. Determine whether the following statement is a tautology: P or $((P \text{ and } (\text{not } Q)) \Rightarrow R)$
4. Is the statement $P \Rightarrow (Q \text{ or } R)$ logically equivalent to the statement $(P \text{ and } (\text{not } Q)) \Rightarrow R$?
Explain why or why not.
5. Exercise 2.1.3 (except part (a))
6. Let $A = \{1, 4, 5, 8\}$, $B = \{3, 5, 9\}$, and $C = \{2, 3, 6, 7\}$.

(a) Draw a Venn diagram showing the relationship between the sets. Label each element.

(b) What are:

- i. $A \cap B$
- ii. $B \cup C$
- iii. $A \cup (B \cap C)$
- iv. $(A \cup B) \cap C$
- v. $A \setminus (B \cap C)$
- vi. $(A \setminus B) \cap C$
- vii. $(B \cup C) \setminus A$
- viii. $\mathcal{P}(B)$

7. Let $A = \{a, b, c, \{a, b, c, d\}, \{c, d, e\}, f, \{f, g\}\}$.

(a) What is $|A|$?

(b) Indicate whether the following statements are true or false:

- i. $\emptyset \in A$
- ii. $f \in A$
- iii. $g \in A$
- iv. $\{f, g\} \in A$
- v. $\{f, g\} \subseteq A$
- vi. $\emptyset \subseteq A$
- vii. $f \subseteq A$
- viii. $\{a, b, c\} \subseteq A$
- ix. $\{a, b, c\} \in A$
- x. $\{a, f\} \subseteq A$
- xi. $\{a, f\} \in A$
- xii. $\{c, g\} \subseteq A$
- xiii. $\{c, g\} \in A$

8. Let $A = (-\infty, -5)$, $B = (-7, 6)$, $C = [4, 10]$, and $U = \mathbb{R}$. What are:

- (a) $A \cap B$
- (b) $A \cup C$
- (c) $A^c \setminus (B \cap C)$
- (d) $(B \cup C) \setminus (A \cup B)^c$
- (e) $A \oplus B \oplus C$
- (f) $C \setminus B^c$

9. Let A , B and C be sets. Prove: $(A \cap B) \times C = (A \times C) \cap (B \times C)$.

10. Consider the statement: $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$ for all sets A, B, C and D .

Is this statement true? If yes, prove it; otherwise show that it is false.