MATH 2320 – Discrete Mathematics Assignment #1
Winter 2016

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Wednesday January 20th in Assignment Box #44.

The following symbols will be used to represent certain sets of numbers:

$\mathbb{Z}$	the set of integers
$\mathbb{N}$	the set of natural numbers, namely $\{1, 2, 3, \ldots\}$
$\mathbb{R}$	the set of real numbers
$\mathbb{Q}$	the set of rational numbers

1. Determine whether the following statements are true, false, or invalid.

   (a) If 3 is even and $4 \geqslant 5$ then $-2^2 = 4$

   (b) If $7 \leqslant 7$ and 4 is even, then $x^2 + 9x - 1 = 0$ has a real solution

   (c) Suppose n is a non-negative integer

   (d) 0 is positive

   (e) If $x \in \mathbb{N}$ then $x = \sqrt{x^2}$

2. For each valid statement in Question 1 that is an implication,

   (a) state the converse of the implication

   (b) determine whether the converse holds

3. State the negation of each of the following statements (assuming that A, B and C are themselves statements with truth values):

   (a) A and (B or not(C))

   (b) A or B or C

   (c) (A and not(B)) or (C or not(D))

Definition. For integers a and b, we say that a divides b (written as "$a \mid b$") if there exists an integer n such that $b = na$. An integer x is said to be even if $x = 2k$ for some integer k. An integer x is said to be odd if $x = 2k + 1$ for some integer k.

4. Prove that each of the following statements is false:

   (a) $9 \mid 33$

   (b) $\forall n \in \mathbb{N}$, $16 \mid n^2$ if and only if $8 \mid n$

   (c) If $x, y \in \mathbb{R}$ such that $x > 0$ and $y > 0$, then $(x + 3)^2 + (y + 4)^2 \leqslant 5^2$

   (d) $\forall x \in \mathbb{R}$, $123x^4 > \dfrac{x^6}{456789}$

5. Rewrite the following statements as English sentences. Also indicate whether each statement is true or false.

   (a) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Q},\ x < y$.

   (b) $\exists x \in \mathbb{Z}, \forall y \in \mathbb{Q},\ x < y$.

6. Find the negation of each statement in Question 5 and indicate whether it is true or false.

7. Let $x, y \in \mathbb{Z}$. Prove that xy is odd if and only if x and y are both odd.

8. Consider the statement: $\forall x \in \mathbb{Z},\ x$ is odd $\Rightarrow 4 \mid (x - x^3)$.

   (a) Is this statement true or false? Justify your answer either with a proof or else with a counter-example.

   (b) What would be the negation of the statement?