

Instructions

- Answer each question completely; justify your answers.
 - This assignment is due at 17:00 on Thursday October 26th in Assignment Box #35.
1. Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be functions. Prove that if $g \circ f$ is surjective and g is injective then f is surjective.
 2. Define $f : (-1, 1) \rightarrow \mathbb{R}$ such that $f(x) = \frac{x}{1-x^2}$.
 - (a) Prove that f is bijective.
 - (b) Prove that there is a one-to-one correspondence between $(-1, 1)$ and $(0, 1)$.
 - (c) Deduce that \mathbb{R} is uncountable.
 3. Exercise 19 of Section 3.3, parts (a) and (d).
 4. Exercise 20 of Section 3.3, except parts (a) and (e).
 5. Is the following statement true or false: if A and B are sets such that $A \subsetneq B$ then $|A| < |B|$. Explain.
 6. Find integers q and r with $0 \leq r < |b|$ such that $a = qb + r$:
 - (a) $a = 129, b = 6$
 - (b) $a = -8141, b = 5$
 - (c) $a = -9162, b = -17$
 7. Let a and b be integers that are not both zero. Prove that $\gcd(a, a + b) = \gcd(a, b)$.
 8. Let $a = 8670$ and $b = 972$. Let g be the greatest common divisor of a and b .
 - (a) Find g .
 - (b) Find integers m and n such that $ma + nb = g$.
 - (c) What is the least common multiple of a and b ?
 9. Let $a = 81876$ and $b = 13191$. Let g be the greatest common divisor of a and b .
 - (a) Find g .
 - (b) Find integers m and n such that $ma + nb = g$.
 - (c) What is the least common multiple of a and b ?