MATH 2320 – Discrete Mathematics Assignment #4
Fall 2017

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Thursday October 12th in Assignment Box #35.

1. Let $A = \{1, 2, 3, \ldots, 13\}$ and define $\preceq$ on A by $a \preceq b$ if a divides b.
   (a) Show that $(A, \preceq)$ is a poset.
   (b) Is the poset totally ordered?
   (c) Does this poset have a maximum? If yes, what is it?
   (d) Does this poset have a minimum? If yes, what is it?
   (e) What are the maximal elements of this poset?
   (f) What are the minimal elements of this poset?
   (g) Draw the poset's Hasse diagram.
2. Let $A = \mathbb{R}^2$ and define $\preceq$ on A by $(a, b) \preceq (x, y)$ if $a \leqslant x$ and $b \geqslant y$.
   (a) Show that $(A, \preceq)$ is a poset.
   (b) Is the poset totally ordered?
   (c) What is the least upper bound on $(9, \sqrt{3})$ and $(-4, 6)$?
   (d) What is the greatest lower bound on $(\pi^2, \frac{4}{5})$ and $(7, -\frac{3}{8})$?
3. Let $f : \mathbb{N} \to \mathbb{Q}$ be defined by $f(x) = \frac{x-1}{x+2}$.
   (a) Prove or disprove: f is surjective.
   (b) Prove or disprove: f is injective.
   (c) Is f bijective?
4. Define $f : \mathbb{N}^2 \to \mathbb{N}$ by $f : (x, y) \mapsto x + y$.
   (a) State the range of f.
   (b) Prove or disprove: f is surjective.
   (c) Prove or disprove: f is injective.
   (d) Is f bijective?
5. Let $A = \{x \in \mathbb{R} \,|\, x \neq -\frac{1}{2}\}$ and define $f : A \to \mathbb{R}$ by $f(x) = \frac{x}{2x+1}$.
   (a) Show that f is injective.
   (b) Is f is surjective?
   (c) What is the range of f?
   (d) Let D be the range of f and define $g : A \to D$ such that $g : x \mapsto f(x)$. What is g^{-1}?
6. Exercise 10 of Section 3.2, except part (a).
7. Exercise 11 of Section 3.2, except part (a).
8. Let $f : A \to B$ and $g : B \to C$ be functions. Prove that if $g \circ f$ is injective then f is injective.