

Instructions

- Answer each question completely; justify your answers.
 - This assignment is due at 17:00 on Thursday October 5th in Assignment Box #35.
1. Consider the statement: $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$ for all sets A, B, C and D .
Is this statement true? If yes, prove it; otherwise show that it is false.
 2. Determine whether the relation \mathcal{R} is reflexive:
 - (a) $\mathcal{R} = \{(x, y) \in \mathbb{Z}^2 \mid x^2 + y^2 \text{ is odd}\}$
 - (b) $\mathcal{R} = \{(x, y) \in \mathbb{Q}^2 \mid xy \geq 0\}$
 3. Determine whether the relation \mathcal{R} is symmetric:
 - (a) $\mathcal{R} = \{(x, y) \in \mathbb{N}^2 \mid x + y = 10\}$
 - (b) $\mathcal{R} = \{(x, y) \in \mathbb{R}^2 \mid x^2 - y^2 \geq 10\}$
 4. Determine whether the relation \mathcal{R} is antisymmetric:
 - (a) $\mathcal{R} = \{(x, y) \in \mathbb{R}^2 \mid x \geq y\}$
 - (b) $\mathcal{R} = \{(x, y) \in \mathbb{R}^2 \mid x^2 \geq y^2\}$
 5. Determine whether the relation \mathcal{R} is transitive:
 - (a) $\mathcal{R} = \{(x, y) \in \mathbb{N}^2 \mid x + y = 10\}$
 - (b) $\mathcal{R} = \{(x, y) \in \mathbb{Q}^2 \mid x + y \in \mathbb{Z}\}$
 6. Define the relation \sim on \mathbb{R}^2 by $(a, b) \sim (c, d)$ if $2a - b = 2c - d$.
 - (a) Prove that \sim is an equivalence relation.
 - (b) Provide a geometric description of $\overline{(2, 4)}$.
 7. Define the relation \sim on \mathbb{Z} by $a \sim b$ if $3a + 5b$ is even.
 - (a) Prove that \sim is an equivalence relation.
 - (b) What is $\bar{3}$?
 - (c) What is \mathbb{Z}/\sim ?