MATH 2320 – Discrete Mathematics Assignment #1
Fall 2017

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Thursday September 21st in Assignment Box #35.

The following symbols will be used to represent certain sets of numbers:

$\mathbb{N}$	the set of natural numbers, namely $\{1, 2, 3, \ldots\}$
$\mathbb{Z}$	the set of integers
$\mathbb{Q}$	the set of rational numbers
$\mathbb{R}$	the set of real numbers
$\mathbb{C}$	the set of complex numbers

1. Determine whether the following are true, false, or not valid statements.
   (a) If (12 is even and $5 \geqslant 5$) then $-1^2 = 1$
   (b) If $k \in \mathbb{N}$ then $x^2 + kx + 1 = 0$ has a real solution
   (c) 0 is not positive
   (d) Suppose n is a non-negative integer
   (e) If $x \in \mathbb{Z}$ then $x = \sqrt{x^2}$

2. For each valid statement in Question 1 that is an implication,
   (a) state the converse of the implication
   (b) determine whether the converse holds

3. State the negation of each of the following statements (assuming that A, B and C are themselves statements with truth values):
   (a) A or (B and not(C))
   (b) A and B and C
   (c) ((not(A)) and not(B)) and (C or not(D))

Definition. For integers a and b, we say that a divides b (written as "$a \mid b$") if there exists an integer q such that $b = qa$. If a does not divide b then we write "$a \nmid b$".

4. Prove that each of the following statements is false:
   (a) $6 \mid 32$
   (b) $\forall n \in \mathbb{N}$, $8 \mid n$ if and only if $4 \mid n^2$
   (c) If $x, y \in \mathbb{R}$ such that $x > 0$ and $y > 0$, then $(x + 5)^2 + (y + 12)^2 \leqslant 13^2$
   (d) $\forall x \in \mathbb{R}$, $123x^4 > \dfrac{x^6}{456789}$

(over)

5. Rewrite the following statements as English sentences. Also indicate whether each statement is true or false.
   (a) $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Q}$ such that $x < y$.
   (b) $\exists x \in \mathbb{Z}$ such that $\forall y \in \mathbb{Q}$, $x \leqslant y$.
6. Find the negation of each statement in Question 5 and indicate whether it is true or false.
7. Let $x, y \in \mathbb{Z}$. Prove that xy is odd if and only if x and y are both odd.
8. Prove: $\forall x \in \mathbb{Z}$, $3 \mid (x^3 - x)$.
9. Consider the statement: $\forall x \in \mathbb{Z}$, x is odd $\Rightarrow 4 \mid (5x - x^3)$.
   (a) Is this statement true or false? Justify your answer either with a proof or else with a counter-example.
   (b) What is the negation of the statement?