

Instructions

- Answer each question completely; justify your answers.
- This assignment is due at 17:00 on Wednesday November 14th in Assignment Box #42.

1. Consider the vectors $v_1 = \begin{bmatrix} 1 \\ 2 \\ -2 \\ 4 \end{bmatrix}$, $v_2 = \begin{bmatrix} 5 \\ 8 \\ 3 \\ 5 \end{bmatrix}$, $v_3 = \begin{bmatrix} 2 \\ 6 \\ -3 \\ 7 \end{bmatrix}$ and $v_4 = \begin{bmatrix} -1 \\ 2 \\ 6 \\ 8 \end{bmatrix}$.

- (a) State a matrix equation, the solution to which reveals whether these vectors are linearly independent or linearly dependent.
- (b) Use Gaussian elimination to solve the equation of part (a) and hence determine whether these vectors are linearly independent or linearly dependent.

2. Let $A = \begin{bmatrix} 2 & -4 \\ -1 & \sqrt{2} \end{bmatrix}$.

- (a) Calculate A^{-1} .
- (b) Calculate A^2 .
- (c) Verify that $(A^{-1})^2 = (A^2)^{-1}$.

3. Let t be a real number and let $A = \begin{bmatrix} \sqrt{t+1} & \sqrt{t} \\ -\sqrt{t} & -\sqrt{t+1} \end{bmatrix}$.

- (a) Calculate A^{-1} .
- (b) Calculate A^{44} .

4. Find X given that $AXB^T = I$, $A = \begin{bmatrix} -3 & 10 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix}$.

5. Solve for X , given that A , B and C are invertible matrices:

- (a) $ABX^{-1}AB = I$
- (b) $ABC^{-1}XB = C$
- (c) $A^2BXB^{-1}C^3 = 2A - B$.

6. Let $A = \begin{bmatrix} 2 & -4 & 1 \\ -1 & 1 & 3 \\ 5 & 2 & -2 \end{bmatrix}$. Find A^{-1} (if it exists).

7. Let $A = \begin{bmatrix} 2 & -4 & 1 & 0 \\ -1 & 0 & 1 & 3 \\ 5 & 2 & 0 & -2 \\ 1 & 2 & 3 & 4 \end{bmatrix}$. Find A^{-1} (if it exists).