The Unruh effect revisited

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Abstract

We give a complete and rigorous proof of the Unruh effect, in the following form. We show that the state of a two-level system, uniformly accelerated with proper acceleration a, and coupled to a scalar bose field initially in the Minkowski vacuum state will converge, asymptotically in the detector's proper time, to the Gibbs state at inverse temperature $\beta = \frac{2\pi}{a}$. The result also holds if the field and detector are initially in an excited state. We treat the problem as one of return to equilibrium, exploiting in particular that the Minkowski vacuum is a KMS state with respect to Lorentz boosts. We then use the recently developed spectral techniques to prove the stated result.

1 Introduction

The following observation, now referred to as the Unruh effect, was made by W. Unruh in 1976 [U]. When a detector, coupled to a relativistic quantum field in its vacuum state, is uniformly accelerated through Minkowski spacetime, with proper acceleration a, it registers a *thermal* black body radiation at temperature $T = \frac{\hbar a}{2\pi c k_{\mathrm{B}}}$. This is the so-called Unruh temperature. In more anthropomorphic terms [UW], "for a free quantum field in its vacuum state in Minkowski spacetime M an observer with uniform acceleration a will feel that he is bathed by a

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thermal distribution of quanta of the field at temperature T." This result has attracted a fair amount of attention and, originally at least, it generated considerable surprise. For a review of various aspects of the subject, nice physical discussions of the phenomenon, and further references, we refer to [Ta], [Wa] and [FuU]. Although the Unruh effect, in the form we treat it in what follows, has acquired generalized acceptance, some scepticism is occasionally still expressed: these alternative viewpoints are critically analyzed in [FuU], where more references can be found.

The reason for the surprise the Unruh effect may generate is that, if you think of the vacuum as "empty space", then you will find it puzzling that a detector, accelerated or not, which may itself initially be in its ground state, will "see particles", since, after all, in the vacuum, there aren't any. In order not to be surprised, one has to remember that, of course, the vacuum is not "empty space", but the ground state of the field, and one should *expect* the detector to react to the presence of the field when it is accelerated through space.

For example, if you were to drag a detector along a non-relativistic chain of oscillators in its ground state, you would certainly expect the coupling between the detector and the oscillators to excite both. The energy for this process is, in final analysis, furnished by the agent that drags the detector along the oscillator chain.

What is nevertheless still surprising in connection with the Unruh effect is the claim that the detector "perceives" a *thermal* distribution of radiation at some particular temperature that only depends on the acceleration. To see what is precisely meant by these statements, it is helpful to get rid of the anthropomorphic terminology used above and in much of the literature as well as of all reference to particles or quanta, which turn out to be irrelevant to the discussion. This is what we will do below. It is worth pointing out in this connection that already in [U], "detection of a particle" is defined by "excitation of the detector", and does therefore not presuppose the actual definition of what a particle precisely is, which is a tricky thing to do, as is well known [Fu]. In fact, the computations in the physics literature of the excitation probability of the detector can be seen to be perturbative computations of the asymptotic state of the detector (see [UW] for example). We therefore adopt the following simple formulation of the Unruh effect. Increasingly precise and rigorous formulations follow below. Consider the coupled detector-field system. Suppose that initially it is in a product state with the field in the vacuum state. Now let the coupled system evolve. At some later (detector proper) time, the state of the system will no longer be a product state. Now trace out the field variables, to obtain the reduced state of the detector (which will be a mixed state, even if the initial state was pure). The Unruh effect states that, asymptotically in the observer's proper time, the latter converges to a (mixed) state, which, to lowest order in the coupling, is the Gibbs state of the uncoupled detector at the aforementioned temperature T. Note that this is not by any means obvious: after all, a priori, it is not clear why the detector state should, asymptotically in time, converge at all, and even if it does, it is not obvious it should tend to a positive temperature state at a particular positive temperature: a priori, it could have been any other mixed state.

It is our goal in this paper to give a complete and rigorous proof of the above statement. We will actually obtain this result from a much stronger statement that we now explain. The way we have formulated the Unruh effect makes it clear already that we think of it as a problem in the theory of open quantum systems in which a small system, here the detector, is coupled to a reservoir, here the field. Let us formulate our result somewhat more precisely. For a completely rigorous statement, we refer to Section 2. The model we consider is the one proposed in [UW], which is itself a simplification of the model considered in [U]. The highly idealized detector is modeled by a two-level system and the field is taken to be a massive or massless Klein-Gordon field. The translational degrees of freedom of the detector are not among the dynamical variables of the theory and the detector follows a prescribed classical trajectory. The two-level system therefore models internal degrees of freedom of the detector. As a result, the observable algebra of the detector is generated by "fermionic" creation/annihilation operators A, $A^\dagger$. The free Heisenberg evolution of the detector is $\dot{A}(\sigma) = -iEA(\sigma)$, where σ is the detector's proper time. In other words, the free detector Hamiltonian is

$$H_{\mathrm{D}} = EA^\dagger A.$$

We note that our results extend without problem to an N-level system, at the cost of irrelevant notational complications and a more involved formulation of the Fermi Golden Rule condition (see (18)).

The coupling between the field and the detector is realized via a monopole, and is ultraviolet regularized; it is sometimes referred to as a de Witt monopole detector (see [Ta]). Suppose initially the detector-field system is in a product state ω_0 with the detector in a state described by some density matrix ρ and the field in the Minkowski vacuum state $|0\rangle$. Let B be a detector observable and F be a field observable and let $\alpha_\sigma^\lambda(BF)$ the Heisenberg evolution of BF under the coupled dynamics, with coupling constant λ. Then we prove that for sufficiently small λ,

$$\omega_\infty^\lambda(BF) := \lim_{\sigma\to\infty} \omega_0(\alpha_\sigma^\lambda(BF))$$

exists for each choice of B and F and and that it equals the thermal equilibrium state of the coupled system at the Unruh temperature (see Theorem 2.2 and 3.5). As a result, to lowest order in the perturbation parameter λ, one finds that this asymptotic state satisfies

$$\omega_\infty^\lambda(BF) = \frac{1}{Z_{\beta,\mathrm{D}}} \mathrm{Tr}\ \mathrm{e}^{-\beta H_{\mathrm{D}}} B\langle 0|F|0\rangle + O(\lambda). \tag{1}$$

Here $\beta = (k_{\mathrm{B}}T)^{-1}$ with T the Unruh temperature and $Z_{\beta,\mathrm{D}} = \mathrm{Tr}\mathrm{e}^{-\beta H_{\mathrm{D}}}$.

Our proof of this result is based on techniques developed in the last decade to prove "return to equilibrium" in open quantum systems [JP1, JP2, BaFS, M1, DJ, DJP]. We combine these with the Bisognano-Wichman theorem [BiWi], which states that the vacuum is a KMS state for the Lorentz boosts on the

Rindler wedge. The relevance of this last result to the Unruh effect (and a generalization to more general spacetimes) was explained a long time ago by Sewell in [Se]. Let us point out that the work of Sewell, together with known stability results of KMS states (see e.g. [Da, KFGV]) imply a result somewhat similar to but considerably weaker than (1), namely

$$\lim_{\sigma\to\infty,\lambda\to 0,\lambda\sigma^2=1}\omega_0(\alpha_\sigma^\lambda(B))=\frac{1}{Z_{\beta,\mathrm{D}}}\mathrm{Tr}\ \mathrm{e}^{-\beta H_\mathrm{D}}B. \tag{2}$$

This is the so-called van Hove weak coupling limit. In our result, the limit $\sigma\to\infty$ is shown to exist for all sufficiently small λ, and to coincide with the right hand side of (1).

The paper is organized as follows. In Section 2, we describe the model in detail and state our main result. We will also comment on the precise role played by the choice of the form factor determining the ultraviolet cutoff in the interaction term. Section 3 is devoted to its proof. The latter uses Araki's perturbation theory for KMS states and its recent extensions, together with the spectral approach to the problem of return to equilibrium developed in the cited references. Since this material is rather technical, we have made an effort to state the result in Section 2 with as little reference to it as possible.

2 The model and the result

We need to give a precise description of the model and in particular of its dynamics. This requires some preliminaries.

2.1 The free field

Let us start by describing in detail the field to which the detector will be coupled. The field operators are represented on the symmetric Fock space $\mathcal{F}$ over $L^2(\mathbb{R}^d,\mathrm{d}\underline{x})$. Here $d\geq 1$ is the dimension of space and $x=(x^0,\underline{x})$ is a point in Minkowski space–time $\mathbb{R}\times\mathbb{R}^d$ (with metric signature $(+,-,\dots,-)$). So $\mathcal{F}:=\oplus_{n\in\mathbb{N}}\mathcal{F}^{(n)}$, where $\mathcal{F}^{(n)}$ is the n-fold symmetric tensor product of the one-particle space $L^2(\mathbb{R}^d,\mathrm{d}\underline{x})$.

Let $\mathcal{S}(\mathbb{R}^{d+1};\mathbb{R})$ and $\mathcal{S}(\mathbb{R}^{d+1};\mathbb{C})$ denote the real and the complex valued Schwartz functions on $\mathbb{R}^{d+1}$, respectively. For $f\in\mathcal{S}(\mathbb{R}^{d+1};\mathbb{C})$, one defines the field operators in the usual way:

$$\Omega=(-\Delta+m^2)^{1/2}, S^\pm f=\int_\mathbb{R}\mathrm{d}t\frac{1}{\sqrt{\Omega}}\mathrm{e}^{\pm\mathrm{i}\Omega t}f_t,\ Q[f]=\frac{1}{\sqrt{2}}(a^\dagger(S^+f)+a(\overline{S^-f})).$$

Here Δ is the Laplacian, $m\geq 0$ the mass, and $a,a^\dagger$ are the usual creation and annihilation operators on $\mathcal{F}$ (we follow the convention that $f\mapsto a^\dagger(f)$ is linear while $f\mapsto a(f)$ is antilinear), and the bar denotes complex conjugation. When

$m = 0$, we will suppose $d > 1$. Writing formally

$$Q[f] = \int_{\mathbb{R}^{d+1}} \mathrm{d}x f(x) Q(x), \quad x = (x^0, \underline{x}),$$

this leads to the familiar

$$Q(x) = \int_{\mathbb{R}^d} \frac{\mathrm{d}\underline{k}}{\sqrt{2\omega(\underline{k})}} \left[\mathrm{e}^{\mathrm{i}\underline{k}\,\underline{x} - \mathrm{i}\omega(\underline{k})x^0} a(\underline{k}) + \mathrm{e}^{-\mathrm{i}\underline{k}\,\underline{x} + \mathrm{i}\omega(\underline{k})x^0} a^*(\underline{k}) \right], \tag{3}$$

where $\omega(\underline{k}) = \sqrt{\underline{k}^2 + m^2}$. The field satsifies the Klein-Gordon equation $\Box Q(x) + m^2 Q(x) = 0$, where $\Box = \partial_{x^0}^2 - \Delta$. We use units in which $\hbar = 1 = c$.

As can be learned in any book on special relativity (such as [Ri]), in an adapted choice of inertial coordinate frame, a uniformly accelerated worldline of proper acceleration $a > 0$, parametrized by its proper time σ, has the form

$$x^0(\sigma) = \frac{1}{a} \sinh a\sigma, \quad x^1(\sigma) = \frac{1}{a} \cosh a\sigma, \quad x^2(\sigma) = 0 = x^3(\sigma).$$

Associated to this worldline is the right wedge (or Rindler wedge) $W_{\mathrm{R}} := \{x \in \mathbb{R}^4 |\ x^1 > |x^0|\}$. It is the intersection of the causal future and past of the worldline, or the collection of spacetime points to which the observer on the worldline can send signals and from which he can also receive signals. Note for later reference that the left wedge $W_{\mathrm{L}} := -W_{\mathrm{R}}$ is the causal complement of $\overline{W}_{\mathrm{R}}$.

There exists a global coordinate system on W_{R} that is particularly well adapted to the description of the problem at hand. It is given by the so-called Rindler coordinates $(\tau, u, \underline{x}_\perp) \in \mathbb{R} \times \mathbb{R}_+^* \times \mathbb{R}^{d-1}$, defined by

$$x^0 = u \sinh \tau, \ x^1 = u \cosh \tau, \ \underline{x}_\perp = (x_2, \dots x_d). \tag{4}$$

Here τ is a global time coordinate on the right wedge. Note that, given $a \in \mathbb{R}^+, (\alpha_2, \dots \alpha_d) \in \mathbb{R}^{d-1}$, the curve $u = 1/a$, $\underline{x}_\perp = (\alpha_2, \dots, \alpha_d)$ is the worldline of a uniformly accelerated observer with proper acceleration a and proper time $\sigma = a^{-1}\tau$. In addition, two points in the right wedge with the same value for the τ-coordinate are considered as simultaneous in the instantaneous rest frame of any such observer (see [Ri]). Among the Lorentz boosts, only the boosts in the x_1-direction leave the right wedge invariant. In inertial coordinates they are given by the linear transformations

$$B_{\tau'} = \begin{bmatrix} \cosh \tau' & \sinh \tau' & 0 & \dots & 0 \\ \sinh \tau' & \cosh \tau' & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}.$$

In the Rindler coordinates, this becomes $B_{\tau'}(\tau, u, \underline{x}_\perp) = (\tau + \tau', u, \underline{x}_\perp)$. In this sense, the boosts in the x_1-direction act as time translations on the Rindler wedge.

Since the field satisfies the Klein-Gordon equation, one has, in Rindler coordinates on $W_{\rm R}$:

$$\left(u^{-2}\partial_\tau^2 - u^{-1}\partial_u u\partial_u + (-\Delta_\perp + m^2)\right) Q(\tau, u, \underline{x}_\perp) = 0. \tag{5}$$

Moreover, the covariance of the free field under the Poincaré group yields, for all $\tau \in \mathbb{R}$,

$$Q[f \circ B_{-\tau}] = \mathrm{e}^{\mathrm{i}L_{\rm F}\tau} Q[f] \mathrm{e}^{-\mathrm{i}L_{\rm F}\tau}, \tag{6}$$

with

$$L_{\rm F} = \mathrm{d}\Gamma(K),\ K = \Omega^{1/2} X^1 \Omega^{1/2}, \tag{7}$$

where X^1 is the operator of multiplication by x^1. In particular, for $x = (\tau, u, \underline{x}_\perp) \in W_{\rm R}$,

$$Q(\tau, u, \underline{x}_\perp) = \mathrm{e}^{\mathrm{i}L_{\rm F}\tau} Q(0, u, \underline{x}_\perp) \mathrm{e}^{-\mathrm{i}L_{\rm F}\tau}.$$

In other words, $L_{\rm F}$ generates the free Heisenberg dynamics of the field operators associated to the right wedge. Let us furthermore introduce, for later purposes, the conjugate field

$$P[f] := \frac{\mathrm{d}}{\mathrm{d}\tau} Q[f \circ B_{-\tau}] \mid_{\tau=0} = \mathrm{i}\left[L_{\rm F}, Q[f]\right]. \tag{8}$$

It then follows from the basic properties of the free field that the equal time commutation relations of the field and the conjugate field are, at $\tau = 0$,

$$[Q(0, u, \underline{x}_\perp), P(0, u', \underline{x}'_\perp)] = \mathrm{i}u\delta_u(u')\ \delta_{\underline{x}_\perp}(\underline{x}'_\perp). \tag{9}$$

The following useful identity follows from (5) and (8):

$$\mathrm{i}\left[L_{\rm F}, P(0, u, \underline{x}_\perp)\right] = -\left(-u\partial_u u\partial_u + u^2(-\Delta_\perp + m^2)\right) Q(0, u, \underline{x}_\perp). \tag{10}$$

For an algebraic formulation of the dynamics, indispensable in what follows, we need to identify the observable algebra of the theory. The observable algebra of the field is $\mathcal{A}_{\rm F} := \{W(f) | f \in \mathcal{S}(\mathbb{R}^{d+1}, \mathbb{R})\}''$, with $W(f) = \mathrm{e}^{-\mathrm{i}Q[f]}$ the usual Weyl operators. One should think of the observable algebra as containing all bounded functions of the (smeared) field operators $Q[f]$ or, more pictorially, all observables that can be constructed from the $Q(x)$, $x \in \mathbb{R}^{d+1}$. Associated to the right and left wedges are local algebras of observables $\mathcal{A}_{\rm F;R,L} := \{W(f) | f \in \mathcal{S}(W_{\rm R,L}, \mathbb{R})\}''$. Again, those should be thought of as containing all observables that can be constructed with the field operators $Q(x)$, for x belonging to the wedge considered. As pointed out above, one can define on $\mathcal{A}_{\rm F}$ an automorphism group α_τ^0 by

$$\alpha^0_{{\rm F},\tau}(A) = \mathrm{e}^{\mathrm{i}L_{\rm F}\tau} A\ \mathrm{e}^{-\mathrm{i}L_{\rm F}\tau}, \qquad A \in \mathcal{A}_{\rm F}. \tag{11}$$

We note that $\alpha^0_{{\rm F},\tau}$ leaves $\mathcal{A}_{\rm F;R}$ invariant.

2.2 The free detector

As pointed out in the introduction, we think of the detector as a two-level system. Its observable algebra is simply the algebra of two by two matrices $\mathcal{B}(\mathbb{C}^2)$. It will be convenient to use a representation of this algebra in which both the ground state and the Gibbs state at inverse temperature β are represented by vectors. This representation, well known in the mathematical physics literature on quantum statistical mechanics, is of course different from the usual one in the standard physics literature in which the latter is represented by a density matrix.

It is defined as follows. One represents the observable algebra $\mathcal{B}(\mathbb{C}^2)$ as $\mathcal{A}_{\rm D} := \mathcal{B}(\mathbb{C}^2) \otimes \mathbb{1}_2$ on $\mathcal{H}_{\rm D} = \mathbb{C}^2 \otimes \mathbb{C}^2$, with in particular $A^\dagger := \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \otimes \mathbb{1}_2$. The algebra $\mathcal{A}_{\rm D}$ is generated by the identity operator, $A^\dagger$, A and $A^\dagger A$ and one has $AA^\dagger + A^\dagger A = \mathbb{1}$.

In this representation, the free Heisenberg evolution of the detector with respect to its proper time σ is generated by the self-adjoint operator $L_{\rm D} := H_{\rm D} \otimes \mathbb{1}_2 - \mathbb{1}_2 \otimes H_{\rm D}$, with

$$H_{\rm D} = \begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix}, \tag{12}$$

for some $E > 0$, where E represents the excitation energy of the detector; $L_{\rm D}$ is referred to as the free Liouvillean of the detector. To see this it is enough to remark that

$$A^\dagger(\sigma) := \alpha^0_{{\rm D},\sigma}(A) := {\rm e}^{{\rm i}L_{\rm D}\sigma} A^\dagger {\rm e}^{-{\rm i}L_{\rm D}\sigma}$$

satisfies the correct Heisenberg equation of motion

$$\dot{A}^\dagger(\sigma) = iEA^\dagger(\sigma) \tag{13}$$

of an unperturbed two-level system. Note that the energy levels of the detector are thought of in this model as pertaining to internal degrees of freedom ([U, UW, Ta]). One should think of the two-level system as being dragged through spacetime by an external agent that ensures it has constant acceleration a. So the translational degrees of freedom of the detector are not dynamical variables in this kind of model. In the representation above, the ground state of the detector can be represented by the vector $|--\rangle$ and the Gibbs state at inverse temperature β by the vector

$$|\beta, {\rm D}\rangle := (1 + {\rm e}^{-\beta E})^{-1/2}(|--\rangle + {\rm e}^{-\beta E/2}|++\rangle) \in \mathcal{H}_{\rm D}.$$

Indeed, one easily checks that, for any $B \in \mathcal{B}(\mathbb{C}^2)$,

$$\langle \beta, {\rm D}|B \otimes \mathbb{1}_2|\beta, {\rm D}\rangle = \frac{1}{Z_{\beta,{\rm D}}} {\rm Tr}\ {\rm e}^{-\beta H_{\rm D}} B, \quad Z_{\beta,{\rm D}} = {\rm Tr}\ {\rm e}^{-\beta H_{\rm D}}.$$

It is the fact that both the ground state and positive temperature states of the detector can be represented by vectors that makes this representation particularly suitable for the problem at hand.

2.3 The uncoupled field-detector system

It is now easy to describe the observable algebra of the joint detector-field system, as well as its uncoupled dynamics. On the Hilbert space $\mathcal{H} := \mathcal{H}_{\rm D} \otimes \mathcal{F}$ we consider the observable algebra $\mathcal{A} := \mathcal{A}_{\rm D} \otimes \mathcal{A}_{{\rm F},R}$ and the self-adjoint operator $L_0 = L_{\rm D} \otimes \mathbb{1}_{\mathcal{F}} + \mathbb{1}_{\mathcal{H}_{\rm D}} \otimes aL_{\rm F}$. The latter determines an automorphism group

$$\alpha^0_\sigma = \alpha^0_{{\rm D},\sigma} \otimes \alpha^0_{{\rm F},a\sigma}$$

of $\mathcal{A}$ in the usual way: $\alpha^0_\sigma(B) = {\rm e}^{{\rm i}L_0\sigma}B{\rm e}^{-{\rm i}L_0\sigma}$, $B \in \mathcal{A}$. Setting $B(\sigma) := \alpha^0_\sigma(B)$ this yields a solution of the Heisenberg equations of motion of the uncoupled detector-field system on the Rindler wedge $W_{\rm R}$, which are given by (5) and (13), with $\tau = a\sigma$.

We will be mostly interested in the state of the system where, initially, the detector is in its ground state, and the field in its Minkowski vacuum. This state is represented by the vector $|{\rm g}\rangle := |--\rangle \otimes |0\rangle \in \mathcal{H}$. We will write, for any $B \in \mathcal{A}$:

$$\langle B\rangle_{\rm g} := \langle {\rm g}|B|{\rm g}\rangle \tag{14}$$

2.4 The coupled field-detector system

For the coupled system we will use the same representation of the observable algebra, but change the dynamics. We will give a precise and mathematically rigorous definition of the dynamics below but to link it with the physics literature on the subject, we start with a formal computation. Let $C(\sigma) = [A(\sigma), A^\dagger(\sigma)]$. According to [UW], the Heisenberg equations of motion of the observables of the coupled system are

$$\begin{aligned} (\Box + m^2)Q(x) &= -\lambda\rho(x_*)(A + A^\dagger)(\frac{\tau(x)}{a}) \\ \dot{A}(\sigma) &= -{\rm i}EA(\sigma) + {\rm i}\lambda C(\sigma)\int {\rm d}u{\rm d}\underline{x}_\perp au\ \rho(x_*)Q(a\sigma, u, \underline{x}_\perp), \end{aligned} \tag{15}$$

The function ρ tunes the coupling between the detector and the field. It is evaluated at

$$x_* := x - x(\tau(x)/a),$$

the spacelike vector linking x in the right wedge to the instantaneous position $x(\sigma)$ of the detector, at proper time $\sigma = \tau(x)/a$, where $\tau(x)$ is the Rindler time coordinate defined in (4).

Let $(\tau, u, \underline{x}_\perp)$ be the Rindler coordinates of the point x then the ones of $x(\sigma)$ are $(\tau, 1/a, \underline{0}_\perp)$ and hence we may identify x_*, whose coordinates are $(0, u - 1/a, \underline{x}_\perp)$, with an element of $(-1/a, +\infty) \times \mathbb{R}^{d-1}$. We take the coupling function ρ to be in $C_0^\infty((-1/a, +\infty) \times \mathbb{R}^{d-1})$, normalized as $\int \rho(\underline{x}){\rm d}\underline{x}^d = 1$. Typically we imagine ρ to be peaked at the origin, so that the field is coupled strongest at the position of the detector. Only for such couplings does it make sense to interpret σ as the proper time of the detector. Indeed, if the detector is coupled to the field over a large spatial region, different parts of the detector undergo a different acceleration and have a different proper time. The mathematical result we obtain then still holds, but does no longer have the same physical interpretation. A coupling *strictly* localized at the position of the detector is formally given by $\rho(\underline{x}) = \delta(\underline{x})$, a situation which does not fit the rigorous mathematical setup presented in this work. We will comment further on the role played by the choice of coupling in Section 2.6.

Using (9) and (10), it is easy to show through a formal computation that the equations (15) are satisfied by the operators $Q^{(\lambda)}(\tau, u, \underline{x}_\perp)$ and $A^{(\lambda)}(\sigma)$ defined as follows:

$$Q^{(\lambda)}(\tau, u, \underline{x}_\perp) := \mathrm{e}^{\mathrm{i}\widetilde{L}_\lambda \frac{\tau}{a}}\ Q(0, u, \underline{x}_\perp)\ \mathrm{e}^{-\mathrm{i}\widetilde{L}_\lambda \frac{\tau}{a}}, \quad A^{(\lambda)}(\sigma) := \mathrm{e}^{\mathrm{i}\widetilde{L}_\lambda \sigma}\ A\ \mathrm{e}^{-\mathrm{i}\widetilde{L}_\lambda \sigma},$$

where

$$\widetilde{L}_\lambda := L_0 + \lambda I, I := (A + A^\dagger) \int \mathrm{d}u \mathrm{d}\underline{x}_\perp au\ \rho(x_*|_{\tau=0}) Q(0, u, \underline{x}_\perp) \tag{16}$$

and $x_*|_{\tau=0}$ is given in Rindler coordinates by $(0, u - 1/a, \underline{x}_\perp)$. In other words, the Liouvillean $\widetilde{L}_\lambda$ generates the correct Heisenberg dynamics of the observables in the representation at hand.

Remark. The analysis we carry out in this paper works for general interactions of the form $I = G \cdot Q(g) + G^* \cdot Q(\overline{g})$, and for sums of such terms, where G are matrices acting on the detector space, and $g \in L^2(\mathbb{R}^3, \mathrm{d}\underline{x})$ are "form factors".

The following result is proved in Section 5.

Proposition 2.1 *The operator $\widetilde{L}_\lambda$ in (16) is for all λ essentially self-adjoint on $D(L_0) \cap D(I)$ and the maps $\alpha^\lambda_\sigma(B) := \mathrm{e}^{\mathrm{i}\widetilde{L}_\lambda \sigma} B \mathrm{e}^{-\mathrm{i}\widetilde{L}_\lambda \sigma}$ with $\sigma \in \mathbb{R}$ and $B \in \mathcal{A}$ define a weakly continuous one-parameter group of automorphisms of the observable algebra $\mathcal{A}$.*

2.5 The result

We are now in a position to give a precise statement of our result. Define

$$g(\varkappa, \underline{k}_\perp) = \Big(\widehat{x_1 \rho(x_*|_{\tau=0})}\Big)\big((|\underline{k}_\perp|^2 + m^2)^{1/2} \sinh \varkappa, \underline{k}_\perp\big), \tag{17}$$

where $\widehat{\ }$ denotes the Fourier transform.

Theorem 2.2 *Let $d \geq 1$ if $m > 0$ and $d \geq 2$ if $m = 0$, and suppose the following "Fermi Golden Rule Condition" holds,*

$$\int_{\mathbb{R}} \mathrm{d}\varkappa\ \mathrm{e}^{-\mathrm{i}\frac{E}{a}\varkappa}\ g(\varkappa, \underline{k}_\perp) \neq 0 \quad \text{for some } \underline{k}_\perp \in \mathbb{R}^{d-1}. \tag{18}$$

Then there is a constant $\lambda_0 > 0$ s.t. if $0 < |\lambda| < \lambda_0$ then

$$\lim_{\sigma \to \infty} \langle \alpha^\lambda_\sigma(B) \rangle_{\mathrm{g}} = \frac{1}{Z_{\beta,\mathrm{D}}} \mathrm{Tr}\ \mathrm{e}^{-\beta H_{\mathrm{D}}} B + O(\lambda^2), \tag{19}$$

for all $B \in \mathcal{B}(\mathbb{C}^2)$, and where $\beta = \frac{2\pi}{a}$.

More generally, if ϱ is any density matrix on $\mathcal{H}$ then

$$\lim_{\sigma\to\infty} \mathrm{Tr}\, \varrho\alpha_\sigma^\lambda(BF) = \Big(\frac{1}{Z_{\beta,\mathrm{D}}}\mathrm{Tr}\ \mathrm{e}^{-\beta H_\mathrm{D}} B\Big) \langle 0|F|0\rangle + O(\lambda), \tag{20}$$

for any detector observable $B \in \mathcal{B}(\mathbb{C}^2)$ and any field observable $F \in \mathcal{A}_\mathrm{F}$.

Result (19) shows that if at $\sigma = 0$ the detector-field system is in a state which is a local perturbation of its ground state, then the reduced density matrix of the detector converges asymptotically in time to the detector's Gibbs state at inverse temperature $\beta = \frac{2\pi}{a}$. This is a (slightly) stronger statement than the formulations usually found in the literature, since it allows both the field and the detector to be initially in an excited state.

Remarks. 1) Theorem 2.2 follows from a more complete result, stated as Theorem 3.5 below, where the l.h.s. of (20) is shown to be equal to the equilibrium state $|\lambda\rangle \in \mathcal{H}$ of the coupled system, see also (28) below. An expansion of $\langle\lambda|\cdot|\lambda\rangle$ for small λ yields the uncoupled equilibrium state plus an error of higher order in λ (the absence of a first order error term in (19 is due to the fact that the expectation of the interaction I in the uncoupled equilibrium state vanishes).

2) The approach to the limit state in (19) is exponentially fast,

$$\big|\mathrm{Tr}\, \varrho\alpha_\sigma^\lambda(B) - \langle\lambda|B|\lambda\rangle\big| < C\|B\|\mathrm{e}^{-\lambda^2\eta\sigma},$$

where C is a constant (depending on the interaction, but not on the initial density matrix ϱ nor on B) and $\eta = (1+\mathrm{e}^{-2\pi E/a})\xi + O(\lambda^2)$, with

$$\xi \equiv \xi(E) = \frac{1}{2a}\int_{\mathbb{R}^{d-1}} \mathrm{d}\underline{k}_\perp \left|\int_{\mathbb{R}} \mathrm{d}\varkappa\ \mathrm{e}^{-\mathrm{i}\frac{E}{a}\varkappa} g(\varkappa, \underline{k}_\perp)\right|^2 \geq 0. \tag{21}$$

The quantity $\tau_\mathrm{relax} = 1/\lambda^2\eta$ is called the *relaxation time* of the process. The purpose of condition (18) is to ensure that $\xi > 0$, i.e., that $\tau_\mathrm{relax} < \infty$. We will show in the following subsection that this is typically the case.

We finally remark that, whereas the leading term of the right hand side of (19) does not depend on the choice of form factor ρ in the interaction term, the relaxation time τ_relax does, via (17) and (21). Nevertheless, we show in the next subsection that τ_relax is independent of the form factor for interactions sharply localized at the position of the detector.

2.6 The Fermi Golden Rule Condition

The goal of this section is to show that (18) is satisfied for "generic" interactions.

Proposition 2.3 *Take the coupling function ρ in (15), (16) to be of the form $\rho(\underline{x}) = \rho_1(x_1)\rho_\perp(\underline{x}_\perp)$, ("square detector") with $\rho_1 \geq 0$. Then condition (18) is satisfied for all E except for $E \in \mathcal{E}$, where $\mathcal{E}$ is a discrete (possibly empty) subset of $\mathbb{R}$. In particular, $\xi(E) > 0$ for all $E \notin \mathcal{E}$.*

Values of E satisfying $\xi(E)=0$ (which form necessarily a subset of $\mathcal{E}$ in the proposition) correspond to energy gaps of the detector Hamiltonian for which thermalization of the detector occurs (if at all) with a larger relaxation time at least of the order λ^{-4} (as opposed to λ^{-2} for E s.t. $\xi(E)>0$), see [M2].

For a particular choice of the coupling function ρ one may resort to a numerical study of the condition (18). On the analytic side we can calculate ξ, (21), in the limit of a strictly localized interaction. More precisely, we choose ρ_1, $\rho_\perp$ as in Proposition 2.3, and consider the family $\rho_\epsilon(\underline{x})=\epsilon^{-d}\rho_1(x_1/\epsilon)\rho_\perp(\underline{x}_\perp/\epsilon)\to\delta(x_1-1/a)\delta(\underline{x}_\perp)$ which represents an interaction localized exactly at the position of the detector in the limit $\epsilon\to 0$. Each ϵ defines thus a $\xi_\epsilon(E)$ by (21), and we obtain, for $d=3$ and $m>0$,

$$\begin{aligned}&\lim_{\epsilon\to 0}\xi_\epsilon(E)\\ &\quad=\ \frac{a}{2}\int_{\mathbb{R}^2}\frac{\mathrm{d}\underline{k}_\perp}{\omega_\perp^4}\left|\int_{\mathbb{R}}\mathrm{d}\varkappa\frac{2\sinh^2\varkappa-\frac{E}{\omega_\perp}\cosh\varkappa-1}{(\frac{E}{\omega_\perp}+\cosh\varkappa)^4}\mathrm{e}^{-\mathrm{i}\left[\frac{E}{a}\varkappa+\frac{\omega_\perp}{a}\sinh\varkappa\right]}\right|^2,\end{aligned}$$

where $\omega_\perp=\sqrt{|\underline{k}_\perp|^2+m^2}$. This limit does not depend on the form of ρ and the leading term of τ_{relax} (as $\epsilon\to 0$) is thus independent of the detector form factor.

Proof of Proposition 2.3. We denote the integral in (18) by $\mathrm{i}\widehat{\rho_\perp}(\underline{k}_\perp)J(E,\omega_\perp)$, where $\omega_\perp=\sqrt{|\underline{k}_\perp|^2+m^2}$, see also (17). For $\omega_\perp\neq 0$ we can make the change of variable $y=\omega_\perp\sinh\varkappa$ to obtain the representation

$$J(E,\omega_\perp)=\int_{\mathbb{R}}\mathrm{d}y\frac{\mathrm{e}^{-\mathrm{i}\frac{E}{a}\operatorname{argsinh}(y/\omega_\perp)}}{\sqrt{\omega_\perp^2+y^2}}f(y),\quad f(y):=\mathrm{e}^{-\mathrm{i}y/a}\Big(\frac{-\mathrm{i}}{a}\widehat{\rho_1}(y)+\widehat{\rho_1}'(y)\Big).\tag{22}$$

We view $\omega_\perp^2=\mu$ in the integral as a parameter, $\mu>0$. We first show that given any $\mu_0>0$, the integral in (22), for $E=0$, does not vanish identically in any neighbourhood of μ_0.

Let us consider $\mu_0=1$; a simple modification of the following argument yields the general case. Assume *ad absurdum* that $J(0,\mu)=0$ for all μ in a neighbourhood of 1. Then, by taking derivaties of $J(0,\mu)$ with respect to μ, at $\mu_0=1$, we see that

$$\int_{\mathbb{R}}\mathrm{d}y\ (1+y^2)^{-n}\,(1+y^2)^{-1/2}f(y)=0,\tag{23}$$

for all $n=0,1,\dots$ Now, it is not difficult to verify that the linear span of all functions $(1+y^2)^{-n}$, $n=1,2,\dots$ is dense in the space of even functions in $L^2(\mathbb{R},dy)$. (One may prove this with little effort via the Fourier transform, for example.) It thus follows from (23) that the even part of f must vanish, $f(y)+f(-y)=0$ for all $y\geq 0$. In particular, $f(0)=0$, which means that

$$a^{-1}=-\mathrm{i}\widehat{\rho_1}'(0)\tag{24}$$

(we assume without loss of generality that ρ_1 is normalized as $\int_{\mathbb{R}}\mathrm{d}x\,\rho_1(x)=1$). On the other hand, we have $-\mathrm{i}\widehat{\rho_1}'(0)=-\int_{\mathbb{R}}\mathrm{d}x\,x\rho_1(x)<a^{-1}$, since in the

integral, $x > -a^{-1}$ due to the fact that ρ_1 is supported in $(-1/a, \infty)$. Therefore condition (24) is not verified.

This shows that given any $\mu_0 > 0$ we can find a $\mu_1 > 0$ (arbitrarily close to μ_0) with the property that $J(0, \mu_1) \neq 0$.

Pick a nonzero $K_0 \in \mathbb{R}^{d-1}$ satisfying $\widehat{\rho_\perp}(K_0) \neq 0$ and set $\mu_0 := \sqrt{|K_0|^2 + m^2}$. Then, by the above argument and by the continuity of $\widehat{\rho_\perp}$ there is a $\mu_1 =: \sqrt{|K_1|^2 + m^2}$ (which is close to μ_0 and defines a K_1 close to K_0) s.t. $J(0, \mu_1) \neq 0$ *and* $\widehat{\rho_\perp}(K_1) \neq 0$. Hence we have shown that there exists a nonzero K_1 satisfying $\mathrm{i}\widehat{\rho_\perp}(K_1) J(0, \omega_1) \neq 0$, where $\omega_1 = \sqrt{|K_1|^2 + m^2}$. Condition (18) is thus satisfied for $E = 0$.

Finally we pass to the other values of E by an analyticity argument. Indeed, one easily sees (best by using the form of J in which one integrates over $\varkappa$ rather than y, c.f. (17), (18)) that the map $E \mapsto J(E, \omega_1)$ is analytic and by the previous argument it does not vanish at $E = 0$. Thus the zeroes of this map are contained in a discrete set $\mathcal{E}(\omega_1) \subset \mathbb{C}$. Any E avoiding this set thus satisfies (18). ■

3 Proof of Theorem 2.2

3.1 Strategy

As mentioned in the introduction, the first ingredient of the proof is the observation that the Minkowski vacuum is (a realization of) the GNS representation of a KMS state on the right wedge algebra for the Lorentz boosts at the inverse temperature $\beta = 2\pi$. This is the content of Theorem 3.1 below. To give a precise statement, we need the so-called modular conjugation operator, defined as follows:

$$J_{\mathrm{F}} = \Gamma(j_{\mathrm{F}}), \text{ where } \forall\, \psi \in L^2(\mathbb{R}^d, \mathrm{d}\underline{x}),\ j_{\mathrm{F}}\psi(\underline{x}) = \overline{\psi}(-x_1, \underline{x}_\perp); \tag{25}$$

here $\Gamma(j)$ stands for the second quantization of j.

Theorem 3.1 ([BiWi]) *The Fock vacuum in $\mathcal{F}$ induces on $\mathcal{A}_{\mathrm{F,R}}$ a state which is KMS at inverse temperature $\beta = 2\pi$ for $\alpha^0_{\mathrm{F},\tau}$. In particular, one has*

$$\mathcal{A}'_{\mathrm{F},R} = \mathcal{A}_{\mathrm{F},L}, \quad J_{\mathrm{F}}\mathcal{A}_{\mathrm{F},R}J_{\mathrm{F}} = \mathcal{A}_{\mathrm{F},L}, \quad \overline{\mathcal{A}_{\mathrm{F},R}|0,\mathrm{F}\rangle} = \mathcal{F} = \overline{\mathcal{A}_{\mathrm{F},L}|0,\mathrm{F}\rangle},$$

and for all $f \in \mathcal{S}(W_{\mathrm{R}}, \mathbb{C})$: $L_{\mathrm{F}}|0,\mathrm{F}\rangle = 0$ and $\mathrm{e}^{-\pi L_{\mathrm{F}}}Q[f]|0,\mathrm{F}\rangle = J_{\mathrm{F}}Q[\overline{f}]|0,\mathrm{F}\rangle$.

This result was proven in considerable generality in [BiWi], for relativistic fields satisfying the Wightman axioms. The result above for the free scalar field can be obtained from essentially direct computations, and we shall not detail it.

Similarly, the states $|\beta, \mathrm{D}\rangle$ introduced in Section 2.2 are GNS representatives of the KMS states at inverse temperature β for the free detector dynamics $\alpha_{\mathrm{D},\sigma}$ on the detector observable algebra $\mathcal{B}(\mathbb{C}^2)$. This well known observation is for convenience summarized in the following lemma. The appropriate conjugate operator is given by

$$J_{\rm D} = E(C \otimes C),$$

where C is the antilinear operator of complex conjugation on $\mathbb{C}^2$ and E is the exchange operator on $\mathbb{C}^2 \otimes \mathbb{C}^2$, $E\varphi \otimes \chi = \chi \otimes \varphi$.

Lemma 3.2 *For any $\beta > 0$, the vector $|\beta, \mathrm{D}\rangle$ induces on $\mathcal{B}(\mathbb{C}^2)$ a state that is KMS at inverse temperature β for $\alpha^0_{\mathrm{D},\sigma}$. In particular, one has*

$$\mathcal{A}'_{\rm D} = \mathbb{1}_2 \otimes \mathcal{B}(\mathbb{C}^2), \quad \mathcal{A}'_{\rm D} = J_{\rm D}\mathcal{A}_{\rm D}J_{\rm D}, \quad \mathcal{A}_{\rm D}|\beta, D\rangle = \mathcal{H} = \mathcal{A}'_{\rm D}|\beta, D\rangle,$$

and

$$\mathrm{e}^{-\beta L_{\rm D}/2}(B \otimes \mathbb{1}_2)|\beta, \mathrm{D}\rangle = J_{\rm D}(B^* \otimes \mathbb{1}_2)|\beta, \mathrm{D}\rangle.$$

Defining, on $\mathcal{H} = \mathcal{H}_{\rm D} \otimes \mathcal{F}$, $J := J_{\rm D} \otimes J_{\rm F}$, it follows that the vector

$$|0\rangle := |\beta = 2\pi/a\rangle \otimes |0, \mathrm{F}\rangle \tag{26}$$

is a GNS representative of the KMS state at inverse temperature $\beta = \frac{2\pi}{a}$ for the free dynamics α^0_σ on $\mathcal{A} = \mathcal{A}_{\rm D} \otimes \mathcal{A}_{\mathrm{F},R}$. This suggests to treat the problem at hand as one of return to equilibrium.

The rest of the argument then proceeds in three steps:

(a) One proves the existence of a GNS representative $|\lambda\rangle \in \mathcal{H}$, defined below, of the KMS state for the perturbed dynamics at the same temperature (Section 3.2);

(b) One reduces the proof of Theorem 3.5 and hence of Theorem 2.2 to showing that the generator of the perturbed dynamics has a simple eigenvalue at 0 and otherwise absolutely continuous spectrum only;

(c) One finally uses spectral deformation theory to prove these two statements.

The strategy in (a)-(b)-(c) has been applied successfully to radiative problems in atomic physics, the spin-boson model, and similar systems in [JP1, JP2, BaFS, M1, DJP], where we refer for further references. A concise introduction to the field can be found in [Pi]. The implementation of this strategy in the present context is reasonably straightforward. We will detail those points that are specific to the current situation.

3.2 Perturbation theory

We define on $\mathcal{H} = \mathcal{H}_{\rm D} \otimes \mathcal{F}$, in addition to $\widetilde{L}_\lambda$ (see 16), the so-called *standard Liouvillean*

$$L_\lambda = \widetilde{L}_\lambda - \lambda JIJ. \tag{27}$$

We outline the proof of the following result in Section 5.

Lemma 3.3 *L_λ is essentially self-adjoint on $D(L_0) \cap D(I) \cap D(JIJ)$ and, for all $B \in \mathcal{A}$,*

$$\alpha^\lambda_\sigma(B) = \mathrm{e}^{\mathrm{i}L_\lambda\sigma} B \mathrm{e}^{-\mathrm{i}L_\lambda\sigma}.$$

A useful feature of the standard Liouvillean (in fact, the motivation for its definition!) is that the unitary it generates leaves the equilibrium state of the coupled system invariant, see (29) below.

Proposition 3.4 *The vector $|0\rangle$ representing the uncoupled equilibrium state, (26), is in the domain of the unbounded operator $\mathrm{e}^{-\frac{\pi}{a}\widetilde{L}_\lambda}$, and the vector*

$$|\lambda\rangle := \frac{\mathrm{e}^{-\frac{\pi}{a}\widetilde{L}_\lambda}|0\rangle}{\|\mathrm{e}^{-\frac{\pi}{a}\widetilde{L}_\lambda}|0\rangle\|} \in \mathcal{H} \tag{28}$$

defines a $(\frac{2\pi}{a}, \alpha^\lambda_\sigma)$-KMS state on $\mathcal{A} = \mathcal{A}_{\mathrm{D}} \otimes \mathcal{A}_{\mathrm{F},R}$ and it satisfies

$$L_\lambda|\lambda\rangle = 0. \tag{29}$$

Proof. To show that $|0\rangle \in \mathrm{Dom}(\mathrm{e}^{-\frac{\pi}{a}\widetilde{L}_\lambda})$ we check that the Dyson series

$$\sum_{n\geq 0}(-\lambda)^n \int_0^{\pi/a} \mathrm{d}t_1 \cdots \int_0^{t_{n-1}} \mathrm{d}t_n\ \alpha^0_{\mathrm{i}t_n}(I)\cdots\alpha^0_{\mathrm{i}t_1}(I)|0\rangle \tag{30}$$

converges. We write the interaction operator conveniently as $I = GQ[g]$, where $G = A + A^\dagger$ and $g(x) = a\delta(x^0)x^1\rho(x_*(\underline{x}))$ has support in W_{R}, c.f. (16). In (30) we have set, for real s,

$$\alpha^0_{\mathrm{i}s}(I) = \mathrm{e}^{-sL_{\mathrm{D}}}G\mathrm{e}^{sL_{\mathrm{D}}}\ \mathrm{e}^{-sL_{\mathrm{F}}}Q[g]\mathrm{e}^{sL_{\mathrm{F}}}.$$

To see that $\mathrm{e}^{-sL_{\mathrm{F}}}Q[g]\mathrm{e}^{sL_{\mathrm{F}}}$ is well defined for $0 \leq s \leq \pi/a$ one shows that since g is supported in the right wedge, the map $t \mapsto \mathrm{e}^{\mathrm{i}tL_{\mathrm{F}}}Q[g]\mathrm{e}^{-\mathrm{i}tL_{\mathrm{F}}} = Q[g \circ B_{-t}]$ has an analytic continuation into the strip $0 < \mathrm{Im}\, t < \pi/a$, and it is continuous at the boundary of the strip ($t \in \mathbb{R}$, $t \in \mathrm{i}\frac{\pi}{a}\mathbb{R}$). This argument is actually part of the proof of the Bisognano–Wichmann theorem, [BiWi]. It follows in particular that the integrals in (30) are well defined and that furthermore

$$\sup_{0\leq \mathrm{Im}\, s\leq \pi/a} \left\|\alpha^0_{\mathrm{i}s}(I)(N+1)^{-1/2}\right\| = C < \infty,$$

where N is the number operator on Fock space. Since $|0\rangle$ is the vacuum on the field part, and each interaction term $\alpha^0_{\mathrm{i}s}(I)$ can increase the particle number by at most one we have the bound $\|\alpha^0_{\mathrm{i}t_n}(I)\cdots\alpha^0_{\mathrm{i}t_1}(I)|0\rangle\| \leq C^n\sqrt{n!}$. It follows that the series (30) converges (for all values of λ) and hence $|0\rangle \in \mathrm{Dom}(\mathrm{e}^{-\frac{\pi}{a}\widetilde{L}_\lambda})$.

The facts that $|\lambda\rangle$ defines a $(\frac{2\pi}{a}, \alpha^\lambda_\sigma)$-KMS state and that $L_\lambda|0\rangle = 0$ follow from Araki's perturbation theory of KMS states, and from perturbation theory of standard Liouville operators, see [DJP]. ■

We are now in a position to state the full result, of which Theorem 2.2 is an immediate consequence:

Theorem 3.5 *Assume that the Fermi Golden Rule Condition (18) is satisfied. There exists λ_0 so that for all $0 < |\lambda| < \lambda_0$, for all density matrices ϱ on $\mathcal{H}$ and for all $B \in \mathcal{A}$*

$$\omega^\lambda_\infty(B) := \lim_{\sigma\to\infty} \mathrm{Tr}\ \varrho\ \alpha^\lambda_\sigma(B) = \langle\lambda|B|\lambda\rangle.$$

Proof. We show in Section 3.3 that the result follows if the spectrum of L_λ is purely absolutely continuous with the exception of a single simple eigenvalue at zero. These spectral characteristics are shown in Theorem 4.2. ■

3.3 Reduction to a spectral problem

We reduce the proof of Theorem 3.5 to a spectral problem via the following simple lemma, which is a variant of the Riemann-Lebesgue lemma:

Lemma 3.6 *Let $\mathcal{H}$ be a Hilbert space, $\phi \in \mathcal{H}$, $\mathcal{A}$ a subalgebra of $\mathcal{B}(\mathcal{H})$ whose commutant we denote by $\mathcal{A}'$, and let L be a self-adjoint operator on $\mathcal{H}$. Suppose that $\mathcal{A}'\phi$ is dense in $\mathcal{H}$, that $\mathrm{e}^{\mathrm{i}L\tau}\mathcal{A}\mathrm{e}^{-\mathrm{i}L\tau} \subset \mathcal{A}$, $\forall\tau$, that $L\phi = 0$, and that on the orthogonal complement of ϕ, L has purely absolutely continuous spectrum.*

Then we have

$$\lim_{\tau\to\infty} \mathrm{Tr}\ \varrho\ \mathrm{e}^{\mathrm{i}L\tau}B\mathrm{e}^{-\mathrm{i}L\tau} = \langle\phi, B\phi\rangle, \tag{31}$$

for all $A \in \mathcal{A}$ and for all density matrices $0 \le \varrho \in \mathcal{L}^1(\mathcal{H})$, $\mathrm{Tr}\varrho = 1$.

Proof. We may diagonalize $\varrho = \sum_{n=1}^{\infty} p_n|\psi_n\rangle\langle\psi_n|$, where $\psi_n \in \mathcal{H}$ and the probabilities $0 \le p_n \le 1$ sum up to one. So it suffices to show (31) for a rank-one density matrix $\varrho = |\psi\rangle\langle\psi|$. Given any $\epsilon > 0$ there is a $B' \in \mathcal{A}'$ s.t. $\|\psi - B'\phi\| < \epsilon$. Thus by replacing ψ by $B'\phi$, commuting B' and $\mathrm{e}^{\mathrm{i}L\tau}A\mathrm{e}^{-\mathrm{i}L\tau}$, and by using the invariance of ϕ under $\mathrm{e}^{-\mathrm{i}L\tau}$ we obtain

$$\mathrm{Tr}\,\varrho\,\mathrm{e}^{\mathrm{i}L\tau}B\mathrm{e}^{-\mathrm{i}L\tau} = \langle\psi, \mathrm{e}^{\mathrm{i}L\tau}B\mathrm{e}^{-\mathrm{i}L\tau}\psi\rangle = \langle\psi, B'\mathrm{e}^{\mathrm{i}L\tau}B\phi\rangle + O(\epsilon), \tag{32}$$

where the remainder is estimated uniformly in τ. Since the spectrum of L is absolutely continuous except for a simple eigenvalue at zero with eigenvector ϕ, the propagator $\mathrm{e}^{\mathrm{i}L\tau}$ converges in the weak sense to the rank-one projection $|\phi\rangle\langle\phi|$, as $\tau \to \infty$. Using this in (32), together with the facts that $\langle\psi, B'\phi\rangle = 1 + O(\epsilon)$, and that ϵ can be chosen arbitrarily small yields relation (31). ■

We apply Lemma 3.6 with $L = L_\lambda$ and $\phi = |\lambda\rangle$. The density of $\mathcal{A}'|\lambda\rangle$ follows from the KMS property of $|\lambda\rangle$, the invariance of $\mathcal{A}$ under $\mathrm{e}^{\mathrm{i}L_\lambda\tau}\cdot\mathrm{e}^{-\mathrm{i}L_\lambda\tau}$ follows from Lemma 3.3 and the relation $L_\lambda|\lambda\rangle = 0$ is shown in Proposition 3.4. It remains to prove that on the orthogonal complement of $|\lambda\rangle$, L_λ has purely absolutely continuous spectrum.

4 Spectral analysis of L_λ

The spectrum of the operator L_{D} consists of two simple eigenvalues $\pm E$ (eigenvectors $|\pm,\mp\rangle$) and a doubly degenerate eigenvalue at 0 (eigenvectors $|\pm,\pm\rangle$). L_{F} has absolutely continuous spectrum covering the entire real axis, and a single embedded eigenvalue at the origin. This eigenvalue is simple and has eigenvector $|0,\mathrm{F}\rangle$. It follows that L_0 has absolutely continuous spectrum covering the axis and three embedded eigenvalues at $0, \pm E$, the one at 0 being doubly degenerate.

Our goal is to show that the nonzero eigenvalues are unstable under the perturbation $\lambda(I - JIJ)$, and that the degeneracy of the eigenvalue zero is lifted. We do this via *spectral deformation theory*, showing that the unstable (parts of the) eigenvalues turn into *resonances* located in the lower complex plane.

4.1 Spectral deformation

For the spectral analysis it is useful to consider the unitarily transformed Hilbert space $L^2(\mathbb{R}^d, \mathrm{d}\varkappa\, \mathrm{d}^{d-1}\underline{k}_\perp)$ of one-particle wave functions of the field, determined by $L^2(\mathbb{R}^d, \mathrm{d}^d\underline{x}) \ni f \mapsto Wf$ with

$$(Wf)(\varkappa, \underline{k}_\perp) := \sqrt{\omega_\perp \cosh\varkappa}\, \widehat{f}(\omega_\perp \sinh\varkappa, \underline{k}_\perp), \tag{33}$$

where $\omega_\perp := \sqrt{|\underline{k}_\perp|^2 + m^2}$ and where $\widehat{f}$ is the Fourier transform of f. The advantage of this representation of the Hilbert space is that the operator K, defined in (7), takes the particularly simple form $K = \mathrm{i}\partial_\varkappa$. The transformation W lifts to Fock space in the usual way. We do not introduce new names for spaces and operators in the transformed system. The Liouville operator (16) is

$$\begin{aligned} L_\lambda &= L_\mathrm{D} + L_\mathrm{F} + \lambda V, \\ L_0 &= L_\mathrm{D} + aL_\mathrm{F}, \quad L_\mathrm{D} = H_\mathrm{D} \otimes \mathbb{1}_2 - \mathbb{1}_2 \otimes H_\mathrm{D}, \quad L_\mathrm{F} = \mathrm{d}\Gamma(\mathrm{i}\partial_\varkappa), \qquad (34) \\ V &= I - JIJ, \quad I = G \otimes \mathbb{1}_2 \otimes \frac{a}{\sqrt{2}} \left\{a^\dagger(g) + a(g)\right\} \qquad (35) \end{aligned}$$

acting on the Hilbert space $\mathcal{H} = \mathbb{C}^2 \otimes \mathbb{C}^2 \otimes \mathcal{F}$, where $\mathcal{F}$ is the bosonic Fock space over $L^2(\mathbb{R}^d, \mathrm{d}\varkappa\, \mathrm{d}^{d-1}\underline{k}_\perp)$. In (35) G is the 2×2 matrix with 0 on the diagonal and 1 on the off-diagonals, and $g(\varkappa, \underline{k}_\perp) = \big(W\Omega^{-1/2} x_1 \rho(x_*|_{\tau=0})\big)(\varkappa, \underline{k}_\perp)$ is given in (17). The action of j_F, (25), is given by $\big(j_\mathrm{F} f\big)(\varkappa, \underline{k}_\perp) = \overline{f}(-\varkappa, \underline{k}_\perp)$.

We describe now the complex deformation. Let $\theta \in \mathbb{R}$. The map

$$\psi_\theta(\varkappa_1, \ldots, \varkappa_n) := \big(U_\theta\psi\big)(\varkappa_1, \ldots, \varkappa_n) := \mathrm{e}^{\mathrm{i}\theta(\varkappa_1 + \cdots \varkappa_n)}\psi(\varkappa_1, \ldots, \varkappa_n)$$

defines a unitary group on $\mathcal{F}$ (we are not displaying the variables $\underline{k}_\perp$ in the argument of ψ since U_θ does not act on them). An easy calculation shows that

$$L_\lambda(\theta) := U_\theta L_\lambda U_\theta^* = L_0(\theta) + \lambda V(\theta), \quad L_0(\theta) = L_0 - a\theta N, \quad V(\theta) = I(\theta) - JI(\theta)J, \tag{36}$$

where $N = \mathrm{d}\Gamma(\mathbb{1})$ is the number operator on $\mathcal{F}$ and

$$I(\theta) = G \otimes \mathbb{1}_2 \otimes \frac{a}{\sqrt{2}} \left\{a^\dagger(\mathrm{e}^{\mathrm{i}\theta\varkappa} g) + a(\mathrm{e}^{\mathrm{i}\overline{\theta}\varkappa} g)\right\}, \tag{37}$$

where we have put the complex conjugate $\overline{\theta}$ in the argument of the annihilation operator in (37) in view of the complexification of θ.

Lemma 4.1 *Let*

$$\theta_0(m,d) := \begin{cases} \infty & \textit{if } m \neq 0 \textit{ and } d \geq 1 \\ \frac{d-1}{2} & \textit{if } m = 0 \textit{ and } d \geq 2 \end{cases} \tag{38}$$

where $m \geq 0$ is the mass of the field and d is the spatial dimension. We have $e^{i\theta\varkappa} W\Omega^{-1/2}h \in L^2(\mathbb{R}^d, d\varkappa d\underline{k}_\perp)$ *for all* $\theta \in \mathbb{C}$ *satisfying* $|\theta| < \theta_0$ *and for all* $h \in \mathcal{S}(\mathbb{R}^d, \mathbb{C})$*. Moreover, for* $|\theta| < \theta_0$*,* $L_\lambda(\theta)$ *is a closed operator on the dense domain* $\mathcal{D} = \mathrm{Dom}(L_0) \cap \mathrm{Dom}(N)$.

Proof. $L_0(\theta)$ is a normal operator, so it is closed. Assume we know that $e^{i\theta\varkappa} W\Omega^{-1/2}\mathcal{S}(\mathbb{R}^d, \mathbb{C}) \subset L^2(\mathbb{R}^d, d\varkappa d\underline{k}_\perp)$, and recall that $x_1\rho(x_*|_{\tau=0}) \in \mathcal{S}(\mathbb{R}^d, \mathbb{C})$. Then, for $\mathrm{Im}\theta \neq 0$ the perturbation $V(\theta)$ is infinitesimally small w.r.t. $L_0(\theta)$, so $L_\lambda(\theta)$ is closed by stability of closedness. For $\mathrm{Im}\theta = 0$ the operator $L_\lambda(\theta)$ is even selfadjoint.

Let $h \in \mathcal{S}(\mathbb{R}^d, \mathbb{C})$. According to (33) we have

$$(W\Omega^{-1/2}h)(\varkappa, \underline{k}_\perp) = \widehat{h}(\omega_\perp \sinh\varkappa, \underline{k}_\perp).$$

Since $\widehat{h} \in \mathcal{S}$ we have that for any integer n there is a constant C_n s.t.

$$\left|\widehat{h}(\omega_\perp \sinh\varkappa, \underline{k}_\perp)\right| < \frac{C_n}{1 + [m^2\sinh^2\varkappa + |\underline{k}_\perp|^2\cosh^2\varkappa]^n}.$$

For $m = 0$ we thus obtain (using an obvious change of variables) the estimate

$$\int_{\mathbb{R}} d\varkappa\, e^{2\theta'|\varkappa|} \int_{\mathbb{R}^{d-1}} d\underline{k}_\perp \left|\widehat{h}(\omega_\perp \sinh\varkappa, \underline{k}_\perp)\right|^2 < \widetilde{C}_n \int_{\mathbb{R}} d\varkappa\, \frac{e^{2\theta'|\varkappa|}}{[\cosh\varkappa]^{d-1}} \tag{39}$$

which is finite provided $\theta' = |\mathrm{Im}\theta| < (d-1)/2$. If $m \neq 0$ then the l.h.s. of (39) is bounded from above by

$$\int_{\mathbb{R}} d\varkappa \int_{\mathbb{R}^{d-1}} d\underline{k}_\perp\, e^{2\theta'|\varkappa|} \frac{C_n^2}{[1/2 + m^2\sinh^2\varkappa]^n[1/2 + |\underline{k}_\perp|^2]^n}$$

which is finite if $\theta' = |\mathrm{Im}\theta| < n$, and n can be chosen arbitrarily large. ■

4.2 Spectra of $L_\lambda(\theta)$ and of L_λ

The goal of this section is to prove the following result.

Theorem 4.2 *Suppose the Fermi Golden Rule Condition (18) holds. There is a* $\lambda_0 > 0$ *s.t. if* $0 < |\lambda| < \lambda_0$ *then the spectrum of* L_λ *consists of a simple eigenvalue at zero and is purely absolutely continuous on the real axis otherwise.*

Remark that the spectrum of $L_0(\theta) = L_0 - a\theta N$ consists of the *isolated eigenvalues* $\pm E$ (simple) and 0 (doubly degenerate), and of the lines of continuous spectrum $\{\mathbb{R} - i\,an\,\mathrm{Im}\theta\}_{n=1,2,\ldots}$. We now analyze the behaviour of the eigenvalues of $L_0(\theta)$ under the perturbation $\lambda V(\theta)$. The strategy is to show that the eigenvalues $\pm E$ are unstable under the perturbation, and that the degeneracy of the eigenvalue zero is lifted. Note that the kernel of L_λ is non-empty by construction, see (29).

Proof of Theorem 4.2. The central part of the proof is the control of the resonances bifurcating out of the eigenvalues $\pm E$ and 0, see Lemma 4.3.

A standard analyticity argument then implies Theorem 4.2. The latter can be summarized as follows: one checks that for all complex z with $\mathrm{Im}z > 0$, $\theta \mapsto \langle \psi_{\overline{\theta}}, (L_\lambda(\theta) - z)^{-1}\phi_\theta\rangle$ is analytic in $0 < |\theta| < \theta_0$, $\mathrm{Im}\theta > 0$, and continuous as $\mathrm{Im}\theta \downarrow 0$, for a dense set of deformation analytic vectors ψ, ϕ (take e.g. finite-particle vectors of Fock space built from test functions $f(\varkappa)$ with compact support). As is well known, the real eigenvalues of $L_\lambda(\theta)$ coincide with those of L_λ, and away from eigenvalues the spectrum of L_λ is purely absolutely continuous.

We now present in more detail the resonance theory.

Lemma 4.3 *Let $\theta' = \mathrm{Im}\theta > 0$. There is a λ_1 (independent of θ) s.t. if $|\lambda| < \lambda_1 \min(1, \theta')$ then in the half-plane $\{\mathrm{Im}z \geq -\theta'/2\}$ the spectrum of $L_\lambda(\theta)$ consists of four eigenvalues $\varepsilon_\pm(\lambda)$ and $\varepsilon_0(\lambda)$ and 0 only (which do not depend on θ).*

Moreover, we have $\varepsilon_j(\lambda) = e_j - \lambda^2\varepsilon_j^{(2)} + o(\lambda^2)$, where $j = +, -, 0$, and

$$\varepsilon_0^{(2)} = \mathrm{i}\,\mathrm{Im}\,\varepsilon_\pm^{(2)} = \mathrm{i}\,(1 + \mathrm{e}^{-2\pi\frac{E}{a}})\xi, \tag{40}$$

whith ξ given in (21).

We remark that the Fermi Golden Rule Condition (18) asserts that $\xi > 0$.

Proof of Lemma 4.3. By an argument of stability of the spectrum it is not difficult to show that the spectrum in the indicated half plane consists of four eigenvalues only.

The position (at second order in λ) is governed by so-called *level shift operators*, see e.g. [M2] and references therein. We explain this with the help of the *Feshbach map* [BaFS].

Let e be an eigenvalue of $L_0(\theta)$ and denote the corresponding (orthogonal) eigenprojection by $Q_e = P_e P_{\mathrm{vac}}$, where P_e is the spectral projection of L_{D} onto e and P_{vac} projects onto the vacuum in $\mathcal{F}$. Set $\overline{Q}_e := \mathbb{1} - Q_e$ and denote by $\overline{X}^e = \overline{Q}_e X \overline{Q}_e \restriction_{\mathrm{Ran}\overline{Q}}$ the restriction of an operator X to $\mathrm{Ran}\overline{Q}$. A standard estimate using Neumann series shows the following fact.

Lemma 4.4 *There is a constant λ_2 (independent of θ) s.t. if $|\lambda| < \lambda_2 \min(E, \theta')$ then, for each eigenvalue e of L_0, the open ball of radius $\theta'/2$ around e, $B(e, \theta'/2)$, belongs to the resolvent set of $\overline{L}_\lambda^e(\theta)$.*

It follows from Lemma 4.4 that the *Feshbach map*

$$F_{e,z}(L_\lambda(\theta)) := Q_e\left(e - \lambda^2 V(\theta)\overline{Q}_e(\overline{L}_\lambda^e(\theta) - z)^{-1}\overline{Q}_e V(\theta)\right)Q_e \tag{41}$$

is well defined for all $z \in B(e, \theta'/2)$. This map has the following remarkable *isospectrality property* [BaFS]: for all $z \in B(e, \theta'/2)$,

$$z \in \mathrm{spec}(L_\lambda(\theta)) \Longleftrightarrow z \in \mathrm{spec}\big(F_{e,z}(L_\lambda(\theta))\big). \tag{42}$$

Thus it suffices to examine the spectrum of the operator $F_{e,z}(L_\lambda(\theta))$ which acts on the finite dimensional space $\mathrm{Ran}Q_e$. We expand the resolvent in (41) around

$\lambda = 0$ and consider spectral parameters $z = e + O(\lambda)$ to obtain

$$F_{e,z}(L_\lambda(\theta)) = Q_e \left(e - \lambda^2 V(\theta)\overline{Q}_e(\overline{L}_0(\theta) - e)^{-1}\overline{Q}_e V(\theta)\right) Q_e + o(\lambda^2),$$

where $\lim_{\lambda\to 0} o(\lambda^2)/\lambda^2 = 0$. We now use analyticity in θ to conclude that

$$F_{e,z}(L_\lambda(\theta)) = Q_e \left(e - \lambda^2 V\overline{Q}_e(\overline{L}_0 - e - \mathrm{i}0_+)^{-1}\overline{Q}_e V\right) Q_e + o(\lambda^2), \qquad (43)$$

where $\mathrm{i}0_+$ stands for the limit of $\mathrm{i}\varepsilon$ as $\varepsilon \downarrow 0$. The operators

$$\Lambda_e := Q_e V\overline{Q}_e(\overline{L}_0 - e - \mathrm{i}0_+)^{-1}\overline{Q}_e V Q_e$$

are called level shift operators. For $e = \pm E$ they reduce in the present case simply to numbers ($\dim \mathrm{Ran} Q_e = 1$), while Λ_0 corresponds here to a 2×2 matrix. Using the expression (35) for V one can calculate explicitly the level shift operators (see also [BaFS, M1, M2] for more details on explicit calculations in related models).

Lemma 4.5 *In the basis $\{|-,-\rangle, |+,+\rangle\}$ of $\mathrm{Ran}Q_0$ we have*

$$\Lambda_0 = \mathrm{i}\xi \mathrm{e}^{-\pi\frac{E}{a}} \begin{bmatrix} \mathrm{e}^{-\pi\frac{E}{a}} & -1 \\ -1 & \mathrm{e}^{\pi\frac{E}{a}} \end{bmatrix}, \quad \textit{and} \quad \mathrm{Im}\Lambda_{\pm E} = (1 + \mathrm{e}^{-2\pi\frac{E}{a}})\xi\,,$$

where ξ is given in (21)

Remark. The Gibbs state of the detector at inverse temperature $\beta = 2\pi/a$ (represented by a vector $\propto [1, \mathrm{e}^{-\pi E/a}]$) spans the kernel of Λ_0.

This lemma together with (43) and the isospectrality (42) shows the expansions (40) and (21). This proves Lemma 4.3, and by the same token, concludes the proof of Theorem 4.2. ∎

5 Proofs of Proposition 2.1 and of Lemma 3.3

Proof of Proposition 2.1. The coupled Liouville opertor (16) has the form $\widetilde{L}_\lambda = L_0 + \lambda I$, where $I = \widetilde{G}Q[\widetilde{g}]$ with $\widetilde{G} = A + A^\dagger$ and $\widetilde{g}(x) = a\delta(x^0)x^1\rho(x_*(x))$ has support in W_R. Essential selfadjointness of $\widetilde{L}_\lambda$ can easily be shown using the Glimm–Jaffe–Nelson commutator theorem, see e.g. [FM], Section 3.

The Araki-Dyson series expansion gives (weakly on a dense set)

$$\begin{aligned} \mathrm{e}^{\mathrm{i}t\widetilde{L}_\lambda} M \mathrm{e}^{-\mathrm{i}t\widetilde{L}_\lambda} \;=\; & \sum_{n=0}^{\infty} \lambda^n \int_0^t \mathrm{d}t_1 \int_{t_1}^t \mathrm{d}t_2 \cdots \int_{t_{n-1}}^t \mathrm{d}t_n \Big[\widetilde{G}(t_1)Q[\widetilde{g}\circ B_{-at_1}], \Big[\cdots \\ & \cdots \Big[\widetilde{G}(t_n)Q[\widetilde{g}\circ B_{-at_n}], \mathrm{e}^{\mathrm{i}tL_0} M \mathrm{e}^{-\mathrm{i}tL_0}\Big] \cdots\Big]\Big], \end{aligned} \qquad (44)$$

where we set $\widetilde{G}(t) = \mathrm{e}^{\mathrm{i}tL_\mathrm{D}} G \mathrm{e}^{-\mathrm{i}tL_\mathrm{D}}$. For $M \in \mathcal{A}$ any element $M' \in \mathcal{A}'$ commutes termwise with the series (44), hence $M'\mathrm{e}^{\mathrm{i}t\widetilde{L}_\lambda} M \mathrm{e}^{-\mathrm{i}t\widetilde{L}_\lambda} = \mathrm{e}^{\mathrm{i}t\widetilde{L}_\lambda} M \mathrm{e}^{-\mathrm{i}t\widetilde{L}_\lambda} M'$. Therefore we have $\mathrm{e}^{\mathrm{i}t\widetilde{L}_\lambda} \mathcal{A} \mathrm{e}^{-\mathrm{i}t\widetilde{L}_\lambda} \in \mathcal{A}$. ∎

Proof of Lemma 3.3. Essential selfadjointness is shown using the Glimm–Jaffe–Nelson commutator theorem, see e.g. [FM] Section 3.1. The fact that $\widetilde{L}_\lambda$ and L_λ define the same dynamics on $\mathcal{A}$ is easily derived by using that $L_\lambda - \widetilde{L}_\lambda$ belongs to the commutant of $\mathcal{A}$ (and e.g. applying the Trotter product formula), see also [FM]. ∎

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