1. Calculate the following subject to the restriction that when factoring, you're only allowed to factor out powers of 2 (so, for example, with the number 60, you're allowed to factor this as $2^2 \cdot 15$, but treat the 15 as though you don't know how (or if) it factors).

   (a) $\left( \dfrac{43}{455} \right)$

   (b) $\left( \dfrac{87}{601} \right)$

   (c) $\left( \dfrac{44}{3323} \right)$

   (d) $\left( \dfrac{5637}{631} \right)$

   (e) $\left( \dfrac{866}{3531} \right)$

   (f) $\left( \dfrac{381}{23} \right)$

   (g) $\left( \dfrac{837}{377} \right)$

   (h) $\left( \dfrac{82001}{643747} \right)$

2. Without identifying any factors of $n$, prove that $n$ is composite.

   (a) $n = 4141$

   (b) $n = 52633$

   (c) $n = 18162001$

   (d) $n = 451149769054931$

3. Let $n \geq 3$ be an odd integer. Prove that if $a \in QR_n$ then $\left( \dfrac{a}{n} \right) = 1$.

4. (a) Let $n$ be an odd composite integer. Prove that at least half of the elements of $\mathbb{Z}_n^*$ are Euler witnesses.

   (b) What proportion of the elements of $\mathbb{Z}_{25}^*$ are Euler witnesses?